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ABSTRACT

Research on error patterns associated with whole number computation is reviewed. Details of the results of some of the individual studies cited are given in the appendices. In Appendix A, 33 addition errors, 27 subtraction errors, 41 multiplication errors, and 41 division errors are identified, and the frequency of these errors made by 352 children in grades three through six is tabulated. Appendix B records the 68 errors in multiplication made by 2100 students in grades four through six. Appendix C lists 57 errors in long division with a one-digit divisor made by 453 children in grades five through eight. In Appendix D, the twelve difficulties most often noted for each of the four operations are identified and responses of 516 children from grades four through eight are tabulated. Appendix E gives examples of six types of errors for addition, subtraction, and multiplication, and four types of errors for division. Appendix F records the persistence of errors made by a group of 221 fourth graders learning division when the divisor was a 2-digit number. Appendix G gives examples of 46 common computation errors which children make in computing with whole numbers. (DT)

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A Review of the Literature on Computational Errors with Whole Numbers

by J.K. Burrows

An individual study project
directed by Dr. D. Robitaille

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Educators have spent many years diagnosing the various computational errors made by students. In 1925 Buswell and Judd stated:

> The term "diagnostic tests" has become very familiar to those working in the field of measurement, and at the present time the chief interest in that field seems to be along the lines of specific analysis and diagnosis of pupils' difficulties (p. 113).

Buswell and Judd (1925) found that at that time, there were thirty-one studies which dealt explicitly with diagnosis and remediation of arithmetical errors. Twenty of those articles were devoted specifically to an analysis of errors, mainly errors in the fundamental processes. Of these studies, a study by Uhl (1917) was the first to diagnose errors by observing the pupil and questioning him while at work. As Buswell and Judd indicate:

> The advantage of this procedure as compared with the method of analyzing test papers can scarcely be overestimated. The great variety of mental processes employed by pupils in doing even very simple work makes it clear that no method can be adequate which fails to get first-hand information with regard to the actual processes used (p. 118).

Of the thirty-one studies, Buswell and Judd found that the best systematic plan of diagnosis was written by Brueckner (1923). Brueckner used Uhl's diagnostic interview technique. He identified ten common errors in addition, seven in subtraction and division and six in multiplication. For Buswell and Judd, Brueckner's study contained the essence of a genuine scheme of diagnosis. However, they saw a need for a more extensive analysis of errors and more detailed examples of errors. They state:

> If such a scheme can be accompanied by a set of diagnostic tests which are so made that they proceed step by step in difficulty and if a sufficiently detailed manual of directions can be supplied for making these diagnoses and for following them with remedial treatment then there is reason to believe that the method of diagnosis and remedial treatment will become a permanent part of the technique of teaching arithmetic (p. 127).

This statement set the stage for the diagnostic studies of Buswell and John (1926). Their work was much more extensive and complete than any study prior to 1926. They individually diagnosed, using Uhl's diagnostic interview technique, 352 students from grades three to six. They identified and gave examples of, and the frequency of occurence of, thirty-three addition errors, twenty-seven subtraction errors, forty-one multiplication errors and forty-one division habits (Appendix A contains the list of habits). They also constructed computational problems that would test for these habits and diagnostic charts on which to record them.

In 1930, Brueckner's text Diagnostic and Remedial Teaching in Arithmetic was published. Brueckner did not introduce any whole number, computational errors unique from those of Buswell and John, however, he did group the errors of Buswell and John into "types of errors". For example, he grouped the thirty-three addition habits under six types: (1) errors in combinations, (2) counting errors, (3) carrying errors, (4) faulty procedures, (5) lapses and other miscellaneous faults, and (6) used scratch paper. Similar groups of errors were constructed in the other three operations.

A study by Burdge (1932) dealing with errors in multiplication revealed some errors not included in the Buswell and John study. Burge again used the individual diagnosis technique. His careful analysis of the mental processes of 2110 children (grades four to six) in their responses to exercises in multiplication revealed sixty-eight errors and questionable habits of work (see Appendix B). In a fashion similar to Brueckner, Burge grouped the habits and errors into seven categories as follows:

(I) Errors and habits in obtaining combinations
(II) Errors and habits in carrying
(III) Irregular errors and habits
(IV) Errors and habits in the misuse of zero
(V) Errors and habits in placing partial products
(VI) Errors and habits in adding partial products
(VII) Miscellaneous errors and habits

Upon comparison of the errors listed by Burge with those of Buswell and John, I have found thirteen multiplication errors not identified in the Buswell and John study. These new errors are identified in the appendix by asterisks.

In his conclusion, Burge supports the diagnostic interview technique with the statement:

> "Relatively few type errors and questionable habits of work could be analyzed with certainty from the written responses." (p. 194).

A very brief exposition by Myers (1935) gives an incomplete treatment of common difficulties in arithmetic; including telling time, zero errors, and number relationships. The article is only worth mentioning as it is the first reference to errors involving zero as a special category of arithmetical errors. The author does not give examples of zero errors, but rather gives examples of questions in which zero errors often occur.

The thirty-fourth yearbook (1935) of the N.S.S.E. was completely devoted to educational diagnosis. Chapter XIV, Diagnosis in Arithmetic, is written by L.J. Brueckner. The list of arithmetical errors in this chapter is a subset of the arithmetical errors cited by Brueckner in his book Diagnostic and Remedial Teaching in Arithmetic (1930).

A study by F.E. Grossnickle (1936) revealed many errors in long division with a one figure divisor. He studied errors made by 453 students in grades five to eight. Before making a detailed analysis of each test paper, Grossnickle constructed a tentative list of errors and faulty

procedures from the studies reported by Buswell and John (1926), Brueckner (1930), Lazar[1] (1928) and Burge (1932). Grossnickle found types of errors in his study not listed in the previous studies. He compiled a list of fifty-seven errors involving division with a one-digit divisor. The errors were grouped under six classifications as follows:

(1) errors of combinations
(2) errors resulting from the use of remainders
(3) errors resulting from zero
(4) errors caused by faulty procedure
(5) errors resulting from lapses of attention
(6) errors resulting from bringing down.

The above list represents a decreasing order of occurence of errors. The errors are listed in Appendix C. Those errors indicated by asterisks are errors that do not appear to be listed in previously mentioned studies. It is interesting to note that Grossnickle found, as did Lazar (1928) that zero was the cause of many errors in long division as revealed in Table V of Appendix C.

Although Grossnickle cited fifty-seven errors in long division, he noted in his summary that many of these errors were infrequent. Errors of combinations in division, multiplication and subtraction, omission of final zero in quotient only, using a remainder greater than the divisor and dropping the remainder when zero was final in the quotient only constituted 59.4% of the total frequencies of errors noted in his study. Thus Grossnickle concludes:

> If a teacher is able to give special consideration to the six types of errors enumerated, provision has been made for almost 60% of the total number of errors which will impede pupil progress in long division with a one-figure divisor (p. 368).

Williams (1937) reports on the results of a study by eleven elementary school principals in Chicago. The prime purpose of the study was to determine the characteristic mathematical deficiencies of children failing

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[1] Lazar's study was unavailable.

in arithmetic. The Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic was administered to 516 children from grades four to eight. Hence no errors apart from those of Buswell and John were listed. This study concentrated on the twelve difficulties most often noted in each process (see Appendix D) and revealed that in each of the processes the leading difficulty resulted from errors in combinations. More than three-fifths of the children were making such errors. Zero errors were also common. In subtraction, 29% of all the children made mistakes because of zeros in the minuend. In multiplication, four types of difficulties with zero appeared among the difficulties with the twelve highest frequencies. In division 28% of the children made errors resulting from the zero difficulty.

In summarizing, Williams states:

> The experience of the Chicago principals, moreover, gives conclusive proof that diagnosis is an individual matter. Group testing shows that pupils are failing to secure correct results, but it does not disclose what specific skill or skills are causing difficulty (p. 600).

F.J. Schonell's textbook Diagnosis of Individual Difficulties in Arithmetic (1937) contains schedules of common errors in the four processes (Appendix E). Schonell does not mention the work of Buswell and John so we must assume that he determined the various error patterns independent of their study. Schonell gives examples of six types of errors in addition, subtraction and division and four in multiplication. Errors listed by Schonell that are unique from those listed by Buswell and John are identified by asterisks. It is interesting to note that there are only three such errors, two in subtraction and one in division.

Schonell is a proponent of the diagnostic interview technique. He states:

> In many cases this information can be derived from a scrutiny of the pupils' work, but there are some instances, particularly with pupils who are very backward in arithmetic, where it is necessary to make observation of their arithmetical habits and to employ oral analysis of their written work (p. 8).

A study by Grossnickle (1939) shows the degree of persistence of errors made by a group of 221 pupils in grade IV who were learning division when the divisor was a two figure number. A record was kept of each pupil's errors on 26 different practice periods in division. Grossnickle found 113 different kinds of errors made on the 26 different practice exercises, but he considered only 24 of the most frequent errors (Appendix F). Two of these 24 errors are unique from errors mentioned in previous studies. These two errors are identified by asterisks.

Following 1939, there was very little research that provides new insight into diagnosis of computational errors involving the whole numbers. There are, however, some studies related to diagnosis of whole number computational errors to which I will make reference.

H. Holland (1942) refers to many types of difficulties encountered in long-division. She mentions such problems as difficulties with division facts, difficulties with subtraction and multiplication, difficulties which arise when the fundamental operations are put in the long-division setting, difficulties with placement of digits, difficulties in quotient estimation and zero difficulties. Holland does not identify any new division errors. Hence, for the purposes of this paper, her article only serves to lend support to the work of Grossnickle (1936, 1939).

Brueckner and Bond (1955) devote chapter eight of their book to diagnosis in arithmetic. The authors refer only to errors listed in the Buswell and John study (1926). Brueckner and Bond acknowledge that many errors are common to all of the operations: namely, errors in number combinations, counting in various ways, faulty procedures and lapses. They realize, as well, that there are special kinds of errors peculiar to each process, such as carrying in addition and multiplication, regrouping in subtraction and remainder difficulties in division. In reference to errors in computation with whole numbers, Brueckner and Bond support, but do not extend the much earlier work of Buswell and John.

A second textbook by F.J. Schonell (1957), Diagnosis and Remedial Teaching in Arithmetic, contains the same schedules of common errors as his earlier textbook (1937). These errors are listed in Appendix E.

A study by Harvey and Kyte (1965) attempted to determine the effects of an instructional program designed to isolate specific errors involving zero in multiplication of whole numbers and to deal with them by remedial teaching. The study was based on a sample of 15 sixth grade classes (517 students). Diagnostic tests were designed and administered in September and again in February. A total of 22 types of multiplication questions involving zero was included in the pre- and post-tests. The tests were not administered individually. After the pre-test, on which the 517 students erred 915 times, teachers were furnished with detailed records of every error made by each pupil in their class. On the basis of the error data, they were to provide both individual and group instruction to meet the ascertained needs of their students. The post-test revealed a

total of 246 errors, a 73.1% decrease in errors from the pre-test. Unfortunately this study gives no indication of specific multiplication errors involving zero, but it does show that adequate remedial work can be organized on the basis of careful observation of errors made by students on class administered tests. As the authors conclude:

> The results show that the program of diagnosis and remediation was very effective. The analysis of the results for all the classes discloses that very few pupils per class failed to correct most of their errors. Stress on the specific needs of specific children led to the desired elimination of most types of zero errors (p. 50).

A publication in School Science and Mathematics by P.C. Burns (1965) stresses the need for good analytical tests that will help in finding what arithmetic difficulties pupils have and why pupils are having the difficulties. Burns advocates the use of the diagnostic interview. His article refers mainly to subtraction of whole numbers. Burns refers to only seven types of errors in subtraction. He does not mention any specific errors in the other operations.

R.B. Ashlock (1972) is also an advocate of the diagnostic interview. Ashlock devotes part of his book to identification of error patterns in computation. He refers to four types of errors in each of the four operations. The errors are similar to errors listed in previous studies. Ashlock offers no new insight into methods of grouping errors in the various operations.

F.K. Reisman (1972) in her book, A Guide to Diagnostic Teaching of Arithmetic, lists forty-six errors children make in computing with whole numbers (Appendix G). Errors indicated by asterisks were not mentioned in previous studies. Reisman, like Ashlock, is an advoate of the diagnostic interview.

L.S. Cox (1975) provides some new insight into the diagnostic process. She advocates giving two tests to the children as follows:

> The initial written test should be given without the teacher's direct supervision. The teacher needs to see how the child performs without the teacher's presence since it is under this condition that the child must ultimately function. The child's paper then should be analyzed for systematic computational errors. Following the analysis and after a tentative diagnosis, the teacher should watch the child work similar problems, being careful to avoid allowing the teacher's presence to alter the child's performance. Many children look to the teacher for non-verbal clues indicating approval or disapproval.
>
> Following this procedure the teacher should ask the child what he was thinking as he worked the problems, or have the child tell what he thinks as he works the problems. This last technique, however, requires that a child be very verbal and many of the children who made systematic errors also have insufficient verbal skills (p. 155).

It is Cox's view that:

> After using the foregoing methods of analysis and observation, the teacher makes a diagnosis regarding the nature of the error (p. 155).

SUMMARY

AND A SUGGESTION FOR FUTURE RESEARCH

To this point, I have reviewed the published literature pertinent to error patterns associated with whole number computation. The studies have revealed a total of 35 unique addition errors, 33 subtraction errors, 54 multiplication errors, and 71 division errors. Some attempts have been made to group the errors into various categories for each operation. It seems that there is a limit to the number of consistent errors that students make in whole number computation: that is, if a student consistently makes a computational error, it is probably one of the 193 computation errors identified in this paper. The Buswell and John study (1926) is the most extensive and identified most of the errors that have been found in my

review of the research. Most of the authors have advocated the diagnostic interview technique in which the student orally describes his method of proceeding through an algorithm. It is probably true that such a technique provides more insight into the actual train of thought or cause of errors than does analysis of a written test. However, very few classroom teachers are trained to prepare and/or administer diagnostic tests and, as a result, there is little real diagnosis and remediation present in elementary school mathematics programmes. Also, the diagnostic interview technique is very time consuming.

Since there seems to be a limit to the number of consistent computational errors, diagnostic tests can be constructed that will provide for as many of these errors as possible. Perhaps the few teachers that are trained to diagnose whole number computational errors and prescribe remedial work, can be put into a more productive setting. Rather than diagnose each student individually, diagnosticians could prescribe remedial work after careful analysis of students' written answers on class administered diagnostic tests. Harvey and Kyte's study as previously mentioned supports this point of view. This type of diagnosis may never be as accurate as the diagnostic interview, but the temporal efficiency of this technique may far outweigh the loss of accuracy. Such an approach implies that the diagnostician need never enter the classroom. The classroom teacher can administer the tests, the diagnostician can diagnose the errors and suggest appropriate areas of remediation for each student or for groups of students. The classroom teacher can then provide the students with the suggested remedial material.

APPENDIX A

TABLE 1

FREQUENCY OF HABITS IN ADDITION (ALL CASES)

Habit	Grade				Total
	III	IV	V	VI	
a1 Errors in combinations	81	103	78	58	320
a2 Counting	61	83	54	17	215
a3 Added carried number last	39	45	45	26	155
a4 Forgot to add carried number	37	38	34	17	126
a5 Retraced work after partly done	26	34	39	22	121
a6 Added carried number irregularly	26	30	28	18	102
a7 Wrote number to be carried	34	25	18	12	89
a8 Carried wrong number	28	19	26	14	87
a9 Irregular procedure in column	16	29	23	18	86
a10 Grouped two or more numbers	25	22	21	16	84
a11 Split numbers	12	29	25	14	80
a12 Used wrong fundamental operation	23	25	20	11	79
a13 Lost place in column	17	17	17	14	65
a14 Depended on visualization	24	8	27	2	61
a15 Disregarded column position	34	11	9	1	55
a16 Omitted one or more digits	13	21	13	5	52
a17 Errors in reading numbers	14	10	21	7	52
a18 Dropped back one or more tens	13	12	17	5	47
a19 Derived unknown combination from familiar one	13	7	11	11	42
a20 Disregarded one column	15	11	8	2	36
a21 Error in writing answer	12	3	14	5	34
a22 Skipped one or more decades	11	7	9	5	32
a23 Carried when there was nothing to carry	6	9	9	5	29
a24 Used scratch paper	7	5	9	0	21
a25 Added in pairs, giving last sum as answer	6	6	6	2	20
a26 Added same digit in two columns	10	6	1	1	18
a27 Wrote carried number in answer	10	2	2	1	15
a28 Added same number twice	4	1	3	3	11
a29 Began with left column	1	1	1	0	3
a30 Confused columns	1	0	0	0	1
a31 Added carried number twice	0	1	0	0	1
a32 Subtracted carried number	0	0	0	1	1
a33 Added imaginary column	0	0	1	0	1
Total number of subjects	96	124	116	78	414

Buswell, G.T. and L. John, Diagnostic Studies in Arithmetic, The University of Chicago: 136-140; 1926.

TABLE 2

FREQUENCY OF HABITS IN SUBTRACTION (ALL CASES)

Habit	Grade				Total
	III	IV	V	VI	
s1 Errors in combinations	62	75	69	40	246
s2 Did not allow for having borrowed	19	50	57	36	162
s3 Counting	43	44	39	10	136
s4 Errors due to zero in minuend	25	39	26	15	105
s5 Said example backward	21	38	29	12	100
s6 Subtracted minuend from subtrahend	47	33	12	4	96
s7 Failed to borrow, gave zero as answer	21	20	14	4	59
s8 Added instead of subtracting	18	9	19	1	47
s9 Error in reading	14	5	13	10	42
s10 Used same digit in two columns	18	15	3	4	40
s11 Derived unknown from known combinations	12	9	13	3	37
s12 Omitted a column	9	13	8	5	35
s13 Deducted from minuend when borrowing was not necessary	2	8	10	5	25
s14 Split numbers	7	5	10	2	24
s15 Used trial-and-error addition	6	7	7	4	24
s16 Ignored a digit	12	6	2	3	23
s17 Deducted 2 from minuend after borrowing	1	5	8	6	20
s18 Error due to minuend and subtrahend digits being same	1	5	10	3	19
s19 Used minuend or subtrahend as remainder	10	6	2	0	18
s20 Reversed digits in remainder	4	7	2	4	17
s21 Confused process with division or multiplication	5	6	3	2	16
s22 Skipped one or more decades	3	4	7	0	14
s23 Increased minuend digit after borrowing	2	2	6	2	12
s24 Based subtraction on multiplication combination	1	2	3	0	6
s25 Error in writing answer	2	1	0	1	4
s26 Began at left column	2	0	1	0	3
s27 Deducted all borrowed numbers from left-hand digit	1	0	1	0	2
Total number of subjects	84	109	109	70	372

TABLE 3

FREQUENCY OF HABITS IN MULTIPLICATION (ALL CASES)

Habit	Grade				Total
	III	IV	V	VI	
m1 Errors in multiplication combinations	36	59	60	41	196
m2 Error in adding the carried number	6	40	58	45	149
m3 Wrote rows of zeros	2	33	40	34	109
m4 Errors in addition	5	31	41	21	98
m5 Carried a wrong number	5	28	40	22	95
m6 Used multiplicand as multiplier	18	33	23	15	89
m7 Forgot to carry	10	30	27	22	89
m8 Error in single zero combinations, zero as multiplier	11	20	23	27	81
m9 Errors due to zero in multiplier	5	26	30	17	78
m10 Used wrong process	18	22	16	10	66
m11 Counted to carry	4	20	28	9	61
m12 Omitted digit in multiplier	1	15	20	16	52
m13 Wrote carried number	8	16	14	9	47
m14 Omitted digit in multiplicand	2	17	12	12	43
m15 Errors due to zero in multiplicand	4	14	15	9	42
m16 Counted to get multiplication combinations	15	11	9	5	40
m17 Error in position of partial products	0	15	15	9	39
m18 Error in single zero combinations, zero as multiplicand	7	13	11	8	39
m19 Confused products when multiplier had two or more digits	1	13	9	9	32
m20 Repeated part of table	3	11	11	6	31
m21 Multiplied by adding	6	11	8	4	29
m22 Did not multiply a digit in multiplicand	5	9	7	7	28
m23 Derived unknown combination from another	3	11	6	6	26
m24 Errors in reading	6	5	11	3	25
m25 Omitted digit in product	0	5	7	5	17
m26 Errors in writing product	2	4	8	2	16
m27 Error in carrying into zero	1	6	7	1	15
m28 Illegible figures	0	3	5	7	15
m29 Forgot to add partial products	0	3	7	2	12
m30 Split multiplier	0	1	6	4	11
m31 Wrote wrong digit of product	0	3	4	2	9
m32 Multiplied by same digit twice	1	1	3	2	7
m33 Reversed digits in product	1	1	2	2	6
m34 Wrote tables	0	0	4	1	5
m35 Used multiplicand or multiplier as product	1	1	1	1	4
m36 Multiplied carried number	2	1	0	1	4
m37 Used digit in product twice	0	1	2	0	3
m38 Added carried number twice	0	1	1	0	2
m39 Carried when there was nothing to carry	0	0	1	0	1
m40 Began at left side	1	0	0	0	1
m41 Multiplied partial products	0	1	0	0	1
Total number of subjects	47	98	102	82	329

TABLE 4

FREQUENCY OF HABITS IN DIVISION (ALL CASES)

Habit	Grade				Total
	III	IV	V	VI	
d1 Errors in division combinations	35	55	59	42	191
d2 Errors in subtraction	4	25	47	37	113
d3 Errors in multiplication	1	20	48	36	105
d4 Used remainder larger than divisor	1	17	39	29	86
d5 Found quotient by trial multiplication	1	8	49	24	82
d6 Neglected to use remainder within example	5	27	25	13	70
d7 Omitted zero resulting from another digit	0	20	22	24	66
d8 Used wrong operation	17	17	24	6	64
d9 Omitted digit in dividend	4	15	27	18	64
d10 Counted to get quotient	5	25	24	4	58
d11 Repeated part of multiplication table	4	15	27	9	55
d12 Used short-division form for long division	0	16	24	10	50
d13 Wrote remainders within example	8	11	17	13	49
d14 Omitted final remainder	4	16	18	11	49
d15 Omitted zero resulting from zero in dividend	3	12	19	12	46
d16 Used long-division form for short division	0	4	27	13	44
d17 Counted in subtracting	3	15	18	6	42
d18 Used too large a product	0	7	21	12	40
d19 Said example backward	9	11	8	7	35
d20 Used remainder without new dividend figure	0	6	14	9	29
d21 Derived unknown combination from known one	1	6	11	8	26
d22 Grouped too many digits in dividend	1	4	12	5	22
d23 Had right answer but used wrong one	0	7	10	4	21
d24 Error in reading	3	2	10	2	17
d25 Used dividend or divisor as quotient	2	4	6	4	16
d26 Reversed dividend and divisor	8	3	2	2	15
d27 Found quotient by adding	1	3	6	4	14
d28 Used digits of divisor separately	0	1	8	1	10
d29 Wrote all remainders at end of example	1	0	6	2	9
d30 Misinterpreted table	0	2	5	2	9
d31 Used digit in dividend twice	0	2	5	2	9
d32 Used second digit of divisor to find quotient	0	1	5	1	7
d33 Began dividing at units' digit of dividend	1	1	4	1	7
d34 Split dividend	0	0	5	1	6
d35 Used endings to find quotient (long division)	0	2	3	1	6
d36 Added remainder to quotient	0	2	1	0	3
d37 Added zeros to dividend when quotient was not a whole number	0	0	1	2	3
d38 Added remainder to next digit of dividend	0	0	0	1	1
d39 Wrote rows of zeros	0	0	1	0	1
d40 Illegible figures	0	0	0	1	1
d41 Dropped zero from divisor and not from dividend.	0	0	0	1	1
Total number of subjects	44	77	103	76	300

APPENDIX B

TABLE 1

ERRORS AND HABITS IN OBTAINING COMBINATIONS IN MULTIPLICATION RECORDED FOR 2,577 PUPILS IN GRADES IV-VI

Error or Habit	Percentage of Pupils in Each Grade					
	Low-fourth (327)*	High-fourth (480)	Low-fifth (453)	High-fifth (473)	Low-sixth (479)	High-sixth (365)
1. Error in combinations	29.1	17.5	33.6	25.2	30.5	21.4
2. Higher combinations unknown	13.2	4.6	17.4	12.3	10.6	8.2
3. Repeats tables from known combination	6.7	5.2	6.4	6.8	6.1	4.7
4. Lower combinations unknown	4.3	0.6	6.0	1.5	1.7	0.5
5. Repeats tables from first	3.7	1.9	3.1	0.8	2.9	0.8
6. Wrong combination fixed	3.4	1.9	3.1	2.5	5.6	3.0
7. Counts from known combination	2.5	2.3	2.9	1.7	5.0	2.2
8. Uses lower combination and addition	2.5	1.9	8.4	11.8	12.3	11.5
9. Sets down columns and adds	1.5	0.6	3.5	2.5	3.1	2.7
10. Depends on tables	0.6	0.4	0.7	0.2	0.4	0.0
11. Makes marks and adds	0.3	0.2	0.9	0.6	0.2	0.5
12. Counts on fingers from known combination	0.3	1.5	1.1	2.1	2.9	3.0
13. Uses special device for nine's	0.0	0.2	0.0	0.0	0.0	0.0
14. Makes dots in adding columns	0.0	0.0	0.4	0.4	0.0	0.5
15. Subtracts from known combination	0.0	0.0	0.2	0.6	1.7	1.1
16. Multiplication by addition	0.0	0.0	0.2	0.2	0.0	0.0

*The numbers in parentheses represent the total number of pupils tested in the grade.

Burge, L.V., "Types of Errors and Questionable Habits of Work in Multiplication," The Elementary School Journal, 32:188-93; Nov. 1932.

TABLE 2

ERRORS AND HABITS IN CARRYING IN MULTIPLICATION RECORDED FOR 2,577 PUPILS IN GRADES IV-VI

Error or Habit	Percentage of Pupils in Each Grade					
	Low-fourth (327)*	High-fourth (480)	Low-fifth (453)	High-fifth (473)	Low-sixth (479)	High-sixth (365)
*17. Sets down carried number	17.1	15.8	24.7	30.0	30.3	35.1
18. Counts in getting carried number	15.6	8.5	9.1	9.5	9.4	8.5
19. Error in carrying	8.9	6.9	11.5	9.3	12.9	6.0
20. Fails to carry	8.0	4.0	4.0	2.7	3.3	3.0
21. Carries wrong number	7.3	5.8	5.7	4.0	6.1	4.4
22. Forms mental image of combinations	2.8	0.8	1.8	4.0	5.8	4.7
23. Counts on fingers to get number carried	2.1	3.1	2.4	2.1	2.7	2.2
24. Puts number to be carried in partial product	0.6	1.0	1.3	0.2	0.8	0.5
25. Reverses number to be set down and carried	0.6	1.0	1.8	0.8	0.6	0.3
26. Sets down combination and carried number and adds	0.3	0.4	0.7	0.6	0.8	1.4
*27. Carries smaller number	0.3	0.0	0.0	0.2	0.2	1.1
28. Secures right combination but sets down wrong number	0.0	1.2	1.1	0.4	0.4	0.8
29. Builds units' digit to tens'	0.0	0.2	2.0	1.9	2.1	3.0
30. Makes dots in adding carried number	0.0	0.0	0.0	0.2	0.6	0.5

*The numbers in parentheses represent the total number of pupils tested in the grade.

TABLE 3

IRREGULAR ERRORS AND HABITS IN MULTIPLICATION RECORDED FOR 2,577 PUPILS IN GRADES IV-VI

Error or Habit	Percentage of Pupils in Each Grade					
	Low-fourth (327)*	High-fourth (480)	Low-fifth (453)	High-fifth (473)	Low-sixth (479)	High-sixth (365)
*31. Multiplies digits in multiplicand by corresponding digits in multiplier	13.1	2.1	3.3	3.2	1.7	1.1
32. Uses multiplicand as multiplier	4.3	3.3	6.6	5.9	5.4	5.2
33. Does not multiply all digits	4.0	2.9	4.6	4.2	5.4	3.3
*34. Can multiply by only one digit	2.8	1.0	1.5	0.0	0.0	0.0
35. Multiplies the carried number	1.8	0.8	0.7	0.2	1.5	0.0
36. Does not multiply by all digits	0.6	2.3	4.9	1.9	1.3	5.2
37. Multiplies by same digit twice	0.6	1.0	0.9	1.1	1.0	0.3
*38. Can multiply by only two digits	0.6	1.2	0.7	2.5	2.1	0.5
*39. Multiplies by carried number	0.0	0.2	0.0	0.4	0.2	0.0
40. In a three-digit multiplier, multiplies by two digits and adds, then by third digit and adds	0.0	0.2	0.7	0.2	0.0	0.0
41. Multiplies backwards	0.0	0.0	0.2	0.0	0.8	0.0

*The numbers in parentheses represent the total number of pupils tested in the grade.

TABLE 4

ERRORS AND HABITS IN THE MISUSE OF ZERO IN MULTIPLICATION
RECORDED FOR 2,577 PUPILS IN GRADES IV-VI

Error or Habit	Percentage of Pupils in Each Grade					
	Low-fourth (327)*	High-fourth (480)	Low-fifth (453)	High-fifth (473)	Low-sixth (479)	High sixth (365)
42. Sets down rows of zeros.....	25.4	31.0	32.0	36.4	38.8	37.5
43. Gives zero value of one in multiplier.....	15.6	12.7	21.4	17.5	17.1	10.4
44. Error in placing partial product when units' digit in multiplier is zero.....	4.6	11.7	15.9	15.4	17.7	24.1
45. Gives zero value of one in multiplicand.....	2.1	1.7	3.8	4.9	3.1	2.2
46. Error in carrying into zero.....	1.2	0.0	1.3	1.5	0.8	0.3
47. Placing partial product when units' digit is zero in product above...	0.6	0.6	0.4	0.0	0.2	0.0
48. Omits units' zero in multiplier...	0.3	1.7	4.0	4.7	3.5	7.1

*The numbers in parentheses represent the total number of pupils tested in the grade.

Table 5

ERRORS AND HABITS IN PLACING PARTIAL PRODUCTS RECORDED FOR 2,577 PUPILS IN GRADES IV-VI

Error or Habit	Percentage of Pupils in Each Grade					
	Low-fourth (327)*	High-fourth (480)	Low-fifth (453)	High-fifth (473)	Low-sixth (479)	High-sixth (365)
49. Error in placing partial product..	4.6	7.1	7.9	5.1	3.5	4.1
50. Keeps columns straight.....	0.3	1.7	0.7	1.1	0.2	0.0
*51. Omits left digit in partial product.....	0.0	0.4	0.4	0.4	0.0	0.5
*52. Double indentation of partial products.....	0.0	0.0	0.4	0.2	0.0	0.0
*53. Indents partial product to right..	0.0	0.0	0.2	0.0	0.2	0.0
*54. Writes two partial products with one-digit multiplier.....	0.0	0.0	0.2	0.2	1.3	0.0

*The numbers in parentheses represent the total number of pupils tested in grade.

TABLE 6

ERRORS AND HABITS IN ADDING PARTIAL PRODUCTS RECORDED FOR 2,577 PUPILS IN GRADES IV-VI

Error or Habit	Percentage of Pupils in Each Grade					
	Low-fourth (327)*	High-fourth (480)	Low-fifth (453)	High-fifth (473)	Low-sixth (479)	High-sixth (365)
55. Error in addition combinations....	3.4	3.7	6.6	6.3	3.1	1.9
56. Counts to get combinations.....	0.9	1.7	1.8	1.9	1.9	0.8
57. Fails to add partial products.....	0.6	0.2	0.0	0.2	0.6	0.3
58. Error in carrying.....	0.6	1.0	1.5	1.3	1.3	1.1
59. Omits digit in adding.....	0.6	0.4	0.2	0.6	0.2	0.0
60. Groups numbers to add.....	0.6	0.8	1.8	2.5	2.3	0.8
*61. Sets down carried number.....	0.3	3.3	7.5	6.1	7.1	12.9
62. Counts on fingers to get combinations.....	0.0	0.4	1.1	1.7	1.7	0.5
63. Makes dots in adding partial products.....	0.0	0.2	1.3	2.9	3.5	3.3
64. Fails to carry.....	0.0	0.0	0.2	0.2	0.0	0.0
*65. Adds diagonally.....	0.0	0.0	0.0	0.2	0.0	0.0

*The numbers in parentheses represent the total number of pupils tested in the grade.

TABLE 7

MISCELLANEOUS ERRORS AND HABITS IN MULTIPLICATION RECORDED FOR 2,577 PUPILS IN GRADES IV-VI

Error or Habit	Percentage of Pupils in each Grade					
	Low-fourth (327)*	High-fourth (480)	Low-fifth (453)	High-fifth (473)	Low-sixth (479)	High sixth (365)
66. Confuses processes of multiplication and addition.....	3.1	1.7	1.5	0.8	0.4	0.0
67. Superfluous use of language.....	3.1	1.0	1.3	0.0	1.3	0.0
*68. Confuses processes of multiplication and subtraction.....	0.3	0.2	0.7	0.2	0.0	0.0

*The numbers in parentheses represent the total number of pupils tested in the grade.

APPENDIX C

TABLE 1

ERRORS OCCURRING IN USE OF REMAINDER FOR GRADES V - VIII

	Error	Grade V	VI	VII	VIII	Total for All Grades
(4)	Used remainder within example greater than divisor	169	156	102	50	477
(5)	Dropped final remainder when zero was final in quotient only	79	58	88	36	261
(6)	Final remainder was two-or-more-figure number	54	68	32	11	165
(7)	Used remainder as a new partial dividend	53	57	29	18	157
(8)	Final remainder was one-figure number equal to or greater than divisor	32	35	28	7	102
(9)	Final remainder correct in example but incorrect in quotient	5	17	10	6	38
*(10)	Final remainder reduced in fraction written as remainder	0	8	12	10	30
*(11)	Last partial dividend written as a remainder	3	10	7	1	21
(12)	Did not find final remainder	7	2	6	2	17
	TOTAL	402	411	314	141	1268

TABLE 2

ERRORS RESULTING FROM ZERO IN GRADES V - VIII

	Error	Grade V	VI	VII	VIII	Total for All Grades
*(13)	Omitted final zero in quotient only	110	101	86	63	360
(14)	Zero final in quotient only, final dividend figure was written for quotient figure	24	19	26	21	90
(15)	Omitted final zero in both dividend and quotient	31	26	13	10	80
(16)	Omitted zero not final in quotient only	18	17	10	9	54
(17)	Extra zeros written in quotient because each remainder treated as a new partial dividend	25	0	0	0	25
*(18)	After zero not final in quotient only, dividend disregarded and zero written in quotient	5	9	5	2	21
*(19)	After zero not final in quotient only, dividend written as remainder.	9	10	2	0	21
*(20)	Omitted zero in the quotient when final in both dividend and quotient, but added zero as a remainder	5	3	6	3	17
*(21)	Zero not final in quotient only, dividend figure written in quotient	2	2	5	4	13
(22)	Zero final in quotient only, extra zero annexed to dividend	0	0	6	6	12
*(23)	Example completed only to zero, not final in quotient only	3	6	0	0	9
*(24)	Remainder not carried to next dividend figure when zero was not final in quotient only	3	0	1	2	6
*(25)	Zero written for remainder when zero was not final in quotient only	4	0	0	0	4
(26)	Last two quotient figures interchanged when zero was final in both dividend and quotient	3	1	0	0	4
(27)	Zero and next quotient figures interchanged when zero was not final in quotient only	0	0	4	0	4
	TOTAL	242	194	164	120	720

Grossnickle, F.E., "Errors and Questionable Habits of Work IN Long Division with a One Figure Divisor," Journal of Educational Research 29: 355-68; Jan. 1936.

TABLE 3

ERRORS RESULTING FROM FACULTY PROCEDURES FOR GRADES V - VIII

	Error	Grade V	Grade VI	Grade VII	Grade VIII	Total for All Grades
*(28)	Placed first quotient figure incorrectly	64	22	3	1	90
*(29)	Found only first and second quotient figures	90	0	0	0	90
*(30)	Began with a two-figure when a one-figure partial dividend was sufficient	36	22	22	6	86
(31)	Used three-figure partial dividend but dropped hundred's figure	45	22	1	0	68
(32)	Used only units' figure of two-figure partial dividend	23	30	0	0	53
*(33)	Duplicated divisor and made it a two-figure number	30	0	0	0	30
*(34)	Multiplied both dividend and divisor by ten then used a two-figure divisor	0	0	0	30	30
(35)	Subtracted larger number from smaller (bottom up)	8	13	6	2	29
(36)	Subtracted larger number from smaller (top down)	3	5	4	5	17
*(37)	Began with second instead of first dividend figure	0	6	3	0	9
*(38)	Began with two-figure partial dividend and wrote two quotient figures for the first estimation	5	1	0	0	6
	TOTAL	304	121	39	44	508

TABLE 4

ERRORS RESULTING FROM LAPSES OF ATTENTION FOR GRADES V - VIII

	Error	Grade V	VI	VII	VIII	Total All Grades
(39)	Used correct multiplication product but wrote incorrect quotient figure	33	34	38	23	128
*(40)	Wrote extra quotient figure or figures	57	30	7	12	106
(41)	Used product too small not detected in subtration	20	30	26	12	88
(42)	Wrote wrong factor of product for quotient figure	13	7	14	6	40
(43)	Did not complete the example	13	9	10	7	39
*(44)	Forgot to multiply quotient figure by divisor	13	13	1	0	27
(45)	Forgot had borrowed in subtraction.	5	2	4	3	14
*(46)	Used last quotient figure for next divisor	1	2	7	4	14
(47)	Misread own figures	2	4	3	1	10
*(48)	Forgot to write quotient figure although used correct product	8	0	2	0	10
	TOTAL	165	131	112	68	476

TABLE 5

ERRORS RESULTING FROM BRINGING DOWN FOR GRADES V - VIII

	Error	Grade V	VI	VII	VIII	Total All Grades
(49)	Final dividend figure not brought down	78	29	27	5	139
(50)	Same number brought down twice	13	23	2	7	45
*(51)	Figure in dividend other than final figure not brought down	19	19	4	2	44
*(52)	Figure not in dividend brought down	4	6	5	1	16
*(53)	Did not bring down dividend figure but wrote 9 for quotient figure	10	3	2	0	15
*(54)	Brought down two figures at once	7	2	2	0	11
*(55)	Figure in quotient brought down	0	3	3	3	9
(56)	Did not bring down zero not final in dividend	2	1	3	1	7
*(57)	Brought down without subtracting for remainder	3	1	1	1	6
	TOTAL	136	87	49	20	292

APPENDIX D

TABLE I

PERCENTAGES OF PUPILS IN GRADE IV B TO GRADE VIII A HAVING DIFFICULTY WITH VARIOUS SKILLS IN ARITHMETIC

Difficulty	Grade IV B	Grade IV A	Grade V B	Grade V A	Grade VI B	Grade VI A	Grade VII B	Grade VII A	Grade VIIIB	Grade VIIIA	All Grades
Addition:											
Errors in combination	80	68	74	81	78	82	64	68	81	80	76
Counting	75	64	74	65	47	54	47	44	50	32	55
Added carried number last	33	32	32	52	57	52	36	42	48	61	45
Forgot to add carried number....	41	32	25	43	24	37	34	24	36	29	32
Repeated work after partly done.	25	36	26	48	41	54	30	18	43	50	37
Added carried number irregularly	14	12	9	20	14	23	11	18	36	23	18
Wrote number to be carried	33	30	19	22	22	25	25	18	29	21	24
Irregular procedure in column ..	29	26	32	31	24	37	32	20	48	32	31
Carried wrong number	18	24	23	28	14	25	34	20	26	21	23
Grouped two or more numbers	14	14	23	15	20	20	13	10	36	21	18
Used wrong fundamental operation	31	26	15	26	8	18	4	6	14	5	15
Lost place in column	25	20	21	31	20	18	10	16	19	27	21
Subtraction:											
Errors in combinations	76	58	60	74	63	62	51	72	50	54	62
Did not allow for having borrowed	33	38	40	56	35	36	47	34	55	37	43
Counting	47	44	47	39	33	30	21	14	26	16	32
Errors due to zero in minuend...	41	38	23	39	22	34	25	26	17	27	29
Said example backwards	29	30	30	24	35	27	13	4	29	20	24
Subtracted minuend from subtrahend	61	44	21	19	22	21	13	4	17	9	23
Failed to borrow; gave zero as answer	24	34	11	24	20	7	2	6	7	2	14
Adding instead of subtracting...	24	22	15	15	10	11	8	4	12	5	12
Error in reading	33	14	17	20	6	11	13	0	7	11	13
Used same digit in two columns..	29	22	13	13	8	9	4	8	2	4	11
Derived unknown from known combination	4	8	2	7	8	5	4	4	5	9	6
Omitted a column	12	12	13	24	16	2	4	4	5	14	11

Multiplication:											
Errors in combinations	82	76	55	74	57	77	58	54	86	37	65
Error in adding the carried number	22	26	30	39	45	66	34	38	74	32	40
Wrote rows of zeros	10	22	32	39	39	55	43	14	50	30	34
Carried a wrong number	22	20	21	48	31	36	15	30	43	27	29
Errors in addition	12	24	40	33	31	45	30	22	40	27	30
Forgot to carry	31	32	34	31	16	34	17	20	21	12	25
Used multiplicand as multiplier.	29	18	17	24	20	23	23	28	43	27	25
Error in single zero combinations, zero as multiplier	18	14	15	41	24	32	15	12	26	20	22
Errors due to zero in multiplier	10	38	30	50	35	48	25	24	21	23	31
Counted to carry	18	12	17	35	27	30	15	18	12	16	20
Omitted digit in multiplier	16	14	8	30	22	30	13	20	12	14	18
Errors due to zero in muliplicand	33	32	23	33	29	12	13	10	21	12	22
Division:											
Errors in combinations	88	76	60	78	61	70	53	54	40	46	63
Errors in subtraction	20	30	30	50	24	57	51	44	45	46	40
Errors in multiplication	8	10	36	48	41	61	45	30	50	50	38
Used remainder larger than divisor	8	18	36	39	35	46	34	18	38	21	29
Found quotient by trial multiplication	8	8	23	31	24	39	32	26	36	39	27
Neglected to use remainder within problem	29	30	23	31	24	14	19	2	12	9	19
Omitted zero resulting from another digit	24	16	32	50	24	37	19	20	29	25	28
Omitted digit in dividend	22	40	28	48	29	32	23	28	24	36	31
Counted to get quotient	24	12	21	20	16	20	13	12	19	5	16
Repeated part of multiplication table	33	32	23	26	25	20	4	10	17	12	20
Used long-division form for short division	4	8	13	37	24	52	19	26	43	34	26
Omitted final remainder	20	36	23	33	27	21	23	22	12	14	23

Williams, C.L. and R.L. Whitaker, "Diagnosis of Arithmetic Difficulties," Elementary School Journal 37: 594-5, April, 1937.

APPENDIX E

COMMON ERRORS IN ADDITION

A. 1. Errors in combinations:

Example (a)

```
 94
 83
 76
 59
311
```

Error: 18 + 4 = 21.
Here the error was a temporary lapse in accuracy.
Combination is really known. Suggested additional
practice with Test X (see Appendix I.)

Example (b)

```
 57
 59
117
```

Error: 9 + 7 = 17.
This proves to be an habitual error for this child, as
revealed by oral testing and by the results of Test 1.
Needs individual practice in plenty of small sums
involving 9 + 7 and 7 + 9.

A. 2. Omitted carrying figure:

Example (a)

```
 39
 48
 77
```

Error: omitted to carry 1.
Here pupil omits carrying number very frequently.
Obviously carrying is not an automatic process. Needs
practice from beginning with simple "carrying" examples.

Example (b)

```
  3
 81
 19
827
 94
1004
```

Error: omitted to carry 2.
Here error due to fact that pupil adds carrying number
in whenever it makes a complete 10, therefore it is
sometimes forgotten.

A. 3. Carried wrong number:

Example

```
 94
 83
 76
 59
302
```

Error: carried 1 instead of 2.
More exercises needed in column addition with a variety
of carrying numbers.

Schonell, F.J., Diagnosis of Individual Difficulties in Arithmetic,
Oliver and Boyd; 1937: 58-63.

A. 4. Added in numbers from other column:

Example

```
  3
 81
 19
827
 94
----
1054
----
```

Error: added in 3 again in tens column.

This is sometimes due to bad setting out of sums, sometimes to lapse of attention.

A. 5. Added in carrying number twice:

Example

```
 28
103
784
  9
----
944
----
```

Error: added in 2 twice in tens column

This is due either to lack of consistency in the time at which the carrying number is added or to retracting steps.

A. 6. Omitted number or numbers from column:

Example

```
 951
 382
 467
 539
 196
----
2455
----
```

Error: omitted 8 in adding.

This is due to losing place in column or to irregular habits of adding. Practice needed in checking answer downwards.

COMMON ERRORS IN SUBTRACTION

S. 1. Omitted to allow for borrowing:

Example

```
 786
  58
----
 738
----
```

Error: omitted to allow for "borrowing," 8 - 5 instead of 8 - 6.

Not a few of these errors appear to be due to lapses, but in the early stages of subtraction many pupils go through a period of intemittent error in this direction. Oral working of examples decreases the error.

S. 2. Subtracted figures in top line (minuend) from those in bottom line (subtrahend):

Example

```
 316
  27
 311
 ---
```

Error: 6-7 and 1-2.

Sometimes this is due to general ignorance of subtraction process, sometimes to nature of figures in the two lines, sometimes to bad teaching. Thus oral examination of one group of backward pupils in a junior school showed that in such an example as the above they would say, "6 take away 7," that is, the top line mentioned first, with the result that in suggestible situations the wrong figures were subtracted.

S.3. Subtraction of like numbers in minuend and subtrahend:

Example (a)

```
 38
  8
 38
 --
```

Error: 8-8 = 8.

Drill required on actual combinations of subtracting like numbers.

Example (b)

```
 250
  49
 211
 ---
```

Error: 5-5 = 1.

*S.4. Subtraction of "0" from a digit or a digit from "0":

Example (a)

```
 890
 889
   9
 ---
```

Error: 0-9 = 9.

This is a common error--that of writing in the answer the number to be subtracted from zero.

Example (b)

```
 607
 318
 319
 ---
```

In general the difficulty seems to be one of adding 10 to 0. The child experiences difficulty in seeing that 10 - 0 = 10, or in then using the 10 from which to subtract.

Example (c)
$$\begin{array}{r} 80 \\ 57 \\ \hline 30 \\ \hline \end{array}$$

S.5. Added instead of subtracting:

Example
$$\begin{array}{r} 387 \\ 196 \\ \hline 421 \\ \hline \end{array}$$

*S.6. "Paying back" to the subtrahend when there was no "borrowing":

Example
$$\begin{array}{r} 987 \\ 832 \\ \hline 145 \\ \hline \end{array}$$
Error: 8-4 instead of 8-3.

Give series of examples alternating "borrowing" and no "borrowing."

COMMON ERRORS IN MULTIPLICATION

M.1. Errors in tables:

Example
$$\begin{array}{r} 7004 \\ \times\ 8 \\ \hline 48034 \\ \hline \end{array}$$
Errors: 4 x 8 = 34, 7 x 8 = 48.

This error far outweighs any other in multiplication. It points to the difficulty and the need of making the basic multiplication facts absolutely automatic, through drill and games.

M.2. Errors in "carrying" numbers:

Example (a)
$$\begin{array}{r} 874615 \\ \times\ 9 \\ \hline 7871495 \\ \hline \end{array}$$
Error: omitted to carry 4

Requires plenty of short sums involving a variety of "carrying."

Example (b)
$$\begin{array}{r} 56 \\ \times\ 90 \\ \hline 4940 \\ \hline \end{array}$$
Error: carried wrong number.

In this case the number written down in the answer was carried.

Example (c)
$$\begin{array}{r} 95347 \\ \times\ 6 \\ \hline 562182 \\ \hline \end{array}$$
Error: 18 + 2 = 21.

Requires practice in combinations involving adding in multiplication.
See Supplementary Test Y.

M.3. Errors in noughts in multiplier or multiplicand:

Example (a)

```
  400
    8
 3288
```

Error: 8 x 0 = 8.

Example (b)

```
   90
   90
 8100
   90
 8190
```

Error: 90 x 0 = 90.

Example (c)

```
   80
  100
  800
   80
   80
  960
```

Error: position of figures, 800 for 8000 and 80 x 0 = 80.

Example (d)

```
  206
   50
10000
```

Error: omitted to carry figure 3 after multiplying 0 by 5.

M.4. Errors in position of figures:

Example (a). Starting to multiply from the right:

```
  34
  22
  68
  68
 136
```

Example (b). Starting to multiply from the left:

```
  52
  31
 156
  52
 208
```

COMMON ERRORS IN DIVISION

D.1. Errors in basic combinations:

Example (a) $\begin{array}{r} 411 \\ 9\overline{)2799} \end{array}$ Error: 27 ÷ 9 = 4

Requires speed practice in division combinations, with and without remainders. Tests 4, 5 and Supplementary Test Z.

Example (b) $\begin{array}{r} 9 \text{ r. } 7 \\ 9\overline{)89} \end{array}$ Error: 89 - 81 = 7.

D.2. Omitted to carry figures:

Example $\begin{array}{r} 32 \\ 4\overline{)138} \end{array}$ Error: omitted to carry 1.

D.3. Remainder larger than divisor:

Example $\begin{array}{r} 6 \text{ r. } 8 \\ 7\overline{)50} \end{array}$ Error: 7 x 6 instead of 7 x 7.

Needs practice with basic combinations involving remainders.

D.4. Omitted "0" from quotient:

Example (a) $\begin{array}{r} 701 \text{ r. } 3 \\ 7\overline{)49010} \end{array}$

Example (b) $\begin{array}{r} 375 \text{ r. } 3 \\ 8\overline{)29643} \end{array}$

*D.5. Carried wrong number:

Example $\begin{array}{r} 632 \text{ r. } 2 \\ 5\overline{)3462} \end{array}$ Error: 6 X 5 = 30, 34 - 30 = 4, then carried 1 instead of 4.

D.6. Used same number in dividend twice:

Example $\begin{array}{r} 3785 \text{ r. } 3 \\ 8\overline{)29643} \end{array}$ Error: 6 used in dividing into 56 and again to divide into 64; 4 used to divide into 64 and again to divide into 43.

APPENDIX F

TABLE 1

NUMBER AND KINDS OF DIFFERENT ERRORS WHICH OCCURRED AT LEAST 52 TIMES AND THE DISTRIBUTION OF THESE ERRORS AMONG THE 26 PRACTICE EXERCISES

Classification and Description of Error	Number of Practice Exercises 1-6	7-13	14-19	20-26	Total
I. Estimation: (Total: 1707)					
1. Estimation too small	415	219	126	139	899
2. Estimation too large	385	116	137	170	808
II. Multiplication: (Total: 2190)					
3. Wrong combination	826	351	143	109	1429
4. Carried wrong numbers	218	154	78	50	500
5. Did not carry when necessary ...	91	74	14	24	203
6. Carried when not necessary	31	13	8	6	58
III. Subtraction: (Total: 1598)					
7. Wrong combination	393	316	86	49	844
8. Forgot he had borrowed	207	197	40	19	463
9. Forgot necessary to borrow	113	93	17	9	232
10. Borrowed when not necessary	21	20	10	8	59
IV. Faculty Procedures: (Total: 764)					
11. Subtracted larger number from smaller (bottom up)	117	71	45	69	302
12. Placed first quotient figure incorrectly	153	23	15	13	204
13. Subtracted larger number from smaller (top down)	29	21	22	46	118
14. Used two-figure partial dividend when three-figure number necessary	35	13	7	16	71
*15. Duplicated the quotient figure .	65	4	0	0	69
V. Remainders: (Total: 696)					
16. Final remainder greater than divisor	82	72	27	46	227
17. Correct remainder in example, incorrect in quotient	57	40	12	5	114
18. Did not find final remainder....	60	33	5	9	107
19. Used remainder within example greater than divisor	16	30	13	35	94
20. Final remainder is equal to divisor	47	15	12	8	82
*21. Found only part of final remainder	52	9	6	5	72
VI. Lapses: (Total: 302)					
22. Used product too small, not detected in subtraction	66	22	12	10	110
23. Did not complete the example ...	54	34	14	4	106
24. Used correct product but wrote wrong quotient figure	66	9	9	2	86
TOTAL	3599	1949	858	851	7257
Percent of Total	49.6	26.9	11.8	11.7	100.0

Grossnickle, F.E., "Constancy of Error in Learning Division with a Two-Figure Divisor," Journal of Educational Research, 33: 191, Nov. 1939.

APPENDIX G

AN ANALYSIS OF SOME COMMON ERRORS CHILDREN MAKE IN ELEMENTARY SCHOOL MATHEMATICS (16)

The following chart includes examples of common errors in elementary school mathematics and analyses of the reasoning for them. This chart will serve as an aid in diagnosing where the child is in need of reteaching.

Analysis	Example
1. Lacks mastery of basic addition combinations.	$\begin{array}{r} 32 \\ +\ 43 \\ \hline 74 \end{array}$
2. Lacks mastery of basic subtraction combinations.	$\begin{array}{r} 38 \\ -\ 25 \\ \hline 12 \end{array}$
3. Lacks mastery of basic multiplication combinations.	$\begin{array}{r} 32 \\ \times\ 3 \\ \hline 86 \end{array}$
4. Lacks mastery of basic division combinations.	$35 \div 5 = 6$ $9\overline{)56}\ 6$ -56 0

Reisman, F.K., A Guide to the Diagnostic Teaching of Arithmetic, The University of Chicago, 1926; 131-7.

Analysis	Example
5. Subtracts incorrectly within the division algorithm.	3) 73 R1 70) 3/230 -21 10 ← - 9 1
6. Error in addition of partial product.	432 X57 3 0 24 21 6 0 24 0 24
7. Does not complete addition:	
a. Does not write regrouped number.	85 + 43 28
b. Leaves out numbers in column addition.	4 8 2 ← + 3 15
8. Rewrites a numeral without computing.	↘ 72 + 15 → 77 → 32 X 3 → 36
9. Does not complete subtraction.	582 - 35 47
10. Does not complete division because of incompleted subtraction.	1) 41 40) 7/3 9 7 -2 8 0 7 7

	Analysis	Example
11.	Fails to complete division stops at first partial quotient.	50 7/370 350
12.	Fails to complete division; leaves remainder greater than divisor.	80 R 9 9/729 720 9
13.	Does not complete multiplication within division algorithm.	1) 201 R 3 200) 3/603 600 3
14.	Does not add by bridging endings--should think. 5 + 9 = 14, so 35 + 9 = 44.	35 + 9 33
*15.	Lacks additive identity concept in addition.	35 +20 50
*16.	Confuses multiplicative identity within addition operation.	71 +13 73
*17.	Lacks additive identity concept in subtraction.	43 -20 20
*18.	Confuses role of zero in subtraction with role of zero in multiplication	37 -20 10
19.	Subtracts top digit from bottom digit whenever regrouping is involved with zero in minuend.	30 -18 28
20.	Confuses role of zero in multiplication with multiplicative identity.	7 X 0 = 7
21.	Confuses place value of quotient by adding extra zero.	20 30/ 60

Analysis	Example
22. Omits zero in quotient	30 R 3 4/ 1203 1200 3
23. Lacks facility with addition algorithm:	
a. Adds ones to ones and tens;	37 + 2 59
b. Adds tens to tens and hundreds;	342 + 36 678
c. Adds ones to tens and hundreds	132 + 6 798
d. Is unable to add horizontally:	345 + 7 + 13 = 185
Thinks: 3 + 7 + 1 = 11; writes 1 4 + 3 = 7 (+ 1 carried) 5 = 5 May add zero to make sum greater than largest addend: 1850	8 5 185
24. Does not regroup ones to tens.	37 +25 52
25. Does not regroup tens to hundreds (or hundreds to thousands).	973 +862 735
26. Regroups when unnecessary.	43 +24 77
27. Writes regrouped tens digit in ones place, carries ones digit (writes the 1 and carries the 2 from "12").	(2) 35 + 7 51

	Analysis	Example
28.	When there are fewer digits in subtrahend:	783
	a. subtracts ones from ones and from tens (and hundreds);	-2 561
	b. subtracts tens from tens and hundreds.	783 -23 560
29.	Does not rename tens digit after regrouping.	54 -9 55
30.	Does not rename hundreds digit after regrouping.	532 -181 451
31.	Does not rename hundreds or tens when renaming ones.	906 -238 778
32.	Does not rename when zero is in tens place, although hundreds are renamed.	803 -478 335
33.	When there are two zeroes in minuend, renames hundred twice but does not rename tens.	5 ~~6~~ ~~7~~00 326 248
34.	Decreases hundreds digit by one when unnecessary.	3 7 1 -1 3 4 1 3 7
35.	Uses ones place factor as addend	32 X 4 126
36.	Adds regrouped number to tens but does not multiply.	35 X 7 65*

* 7 X 5 = 35;
30 + 30 = 60

Analysis	Example
37. Multiplies digits within one factor. *4 X 1 = 4; 1 X 30 = 30	31 X 4 34*
38. Multiplies by only one number	457 X 12 914
39. "Carries" wrong number	8 67 X40 3220
40. Does not multiply ones times tens	32 X24 648
41. Reverses divisor with dividend. *Thinks 6 ÷ 3 instead of 30 ÷ 6	2 * 6/30
42. Does not regroup; treats each column as separate addition e.ample	23 + 8 211
43. Subtracts smaller digit from larger at all times to avoid renaming	273 -639 446
44. Does not add regrouped number	37 X 7 219
45. Confuses place value in division:	1) 200) 201 3/6003 6000 3
a. Considers thousands divided by ones as hundreds divided by ones;	3

Analysis	Example
b. records partial quotient as tens instead of ones;	50) 100) 150 7/735 -700 35 35
c. omits zero needed to show no ones in quotient.	2 R 1 3/ 61 6 1

46. Ignores remainder because:

a. does not complete subtraction;

b. does not see need for further computation;

c. does not know what to do with "2" if subtraction occurs, so does not compute further.

Example
80 7/ 562 560

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